

Topics we offer

IMIS REU Program in Combinatorics

Xidian University

2026-2027

1.

Extremal incidence problems

Given a set of n points in the real affine plane such that no line contains more than k of the points, how few lines can there be that contain at least 2 of the points? How few lines can there be that contain exactly 2 of the points? These are classical questions in combinatorial geometry that were considered by Erdős and many others. In this project, we will consider variants of these questions in higher dimensions, and for point sets contained in affine geometries over other fields (besides \mathbb{R}).

KEY REFERENCES

- R. Campbell, J. Geelen & M. E. Kroeker, Average plane-size in complex-representable matroids, *Combinatorica*, **45** (2025), 53.
- Z. Dvir, Incidence theorems and their applications, *Foundations and Trends in Theoretical Computer Science*, **6** (2012), 257–393.
- J. G. Oxley, *Matroid Theory*, Oxford Graduate Texts in Mathematics, **3**, Oxford University Press, Oxford, 2006.

Advisor **Prof. Ben Lund**

2.

Distance problems

How many distinct distances can n points in the plane determine? This classical question, posed by Erdős in 1946, has driven remarkable progress at the intersection of combinatorics, incidence geometry, and algebraic geometry. In this project, we will study variants of this problem and applications.

KEY REFERENCES

- P. Erdős, On sets of distances of n points, *The American Mathematical Monthly*, **53** (1946), 248–250.
- L. Guth & N. H. Katz, On the Erdős distinct distances problem in the plane, *Annals of Mathematics*, **2**(181) (2015), 155–190.
- B. Murphy, G. Petridis, T. Pham, M. Rudnev & S. Stevens, On the pinned distances problem in positive characteristic, *Journal of the London Mathematical Society*, **2**(105) (2022), 469–499.

Advisor **Prof. Thang Pham**

3.

Restriction theory

Restriction theory studies how the Fourier transform of a function behaves when restricted to algebraic varieties. In this project, we will focus on restriction and extension problems over finite fields, especially for varieties such as paraboloids, cones, and spheres. This is a challenging topic, so the first goal will be to understand the main problems and techniques in the area. We will then explore several extensions motivated by applications in additive combinatorics and incidence geometry.

KEY REFERENCES

- C. González-Riquelme & D. Oliveira e Silva, Sharp restriction theory over finite fields, *London Mathematical Society Newsletter*, **518** (2026), 25–30.
- D. Koh, S. Lee & T. Pham, On the finite field cone restriction conjecture in four dimensions and applications in incidence geometry, *International Mathematics Research Notices*, **2022** (2022), 17079–17111.
- D. Koh, T. Pham & L. A. Vinh, Extension theorems and a connection to the Erdős-Falconer distance problem over finite fields, *Journal of Functional Analysis*, **281** (2021), 109137.
- M. Lewko, Finite field restriction estimates based on Kakeya maximal operator estimates, *Journal of the European Mathematical Society*, **21** (2019), 3649–3707.
- G. Mockenhaupt & T. Tao, Restriction and Kakeya phenomena for finite fields, *Duke Mathematical Journal*, **121** (2004), 35–74.
- M. Rudnev & I. D. Shkredov, On the restriction problem for discrete paraboloid in lower dimension, *Advances in Mathematics*, **339** (2018), 657–671.

Advisor **Prof. Thang Pham**

4.

Selected Topics in Additive Combinatorics

A central result in this area is the Erdős–Szemerédi sum-product theorem, which shows that a finite set cannot be highly structured with respect to both addition and multiplication at the same time. This phenomenon has led to deep developments in arithmetic combinatorics, incidence geometry, and geometric measure theory. In this project, we will study variants of the sum-product problem and explore applications to related questions, including expanding polynomials, distinct distances, and Falconer-type problems.

KEY REFERENCES

- P. Erdős & E. Szemerédi, On sums and products of integers, in *Studies in Pure Mathematics: To the Memory of Paul Turán*, Birkhäuser, Basel, 1983, 213–218.
- D. Koh, T. Pham & C.-Y. Shen, Falconer type functions in three variables, *Journal of Functional Analysis*, **286** (2024), 110246.
- T. Pham, L. A. Vinh & F. de Zeeuw, Three-variable expanding polynomials and higher-dimensional distinct distances, *Combinatorica*, **39** (2019), 411–426.
- T. Tao, Expanding polynomials over finite fields of large characteristic, and a regularity lemma for definable sets, *Contributions to Discrete Mathematics*, **10** (2015), 22–98.

Advisor **Prof. Thang Pham**

COMBINATORICS

5.

Factoring groups into subsets

The factorization theory of abelian groups deals with decomposing an abelian group into a direct sum of its subsets. In this project, we will study some properties of such group decompositions.

KEY REFERENCES

S. Szabó & A. D. Sands, Factoring Groups into Subsets, *Lecture Notes in Pure and Applied Mathematics*, **257**, CRC Press, Boca Raton, FL, 2009.

Advisor **Prof. Tao Zhang**

ADDITIVE COMBINATORICS

6.

Zero-sum Problems

A typical direct zero-sum problem studies conditions which ensure that given sequences have nonempty zero-sum subsequences with prescribed properties. The associated inverse zero-sum problem studies the structure of extremal sequences which have no such zero-sum subsequences. For example, the famous Erdős-Ginzburg-Ziv Theorem says that any sequence of $2n-1$ integers has a subsequence of n elements, with their sum being a multiple of n . In this project, we will investigate some invariants of classical zero-sum invariants.

KEY REFERENCES

- W. Gao & A. Geroldinger, Zero-sum problems in finite abelian groups: a survey, *Expositiones Mathematicae*, **24** (2006), 337–369.
- D. J. Grynkiewicz, *Structural Additive Theory*, Developments in Mathematics, **30**, Springer, Cham, 2013.

Advisor **Prof. Qinghai Zhong**